

Electrodynamics & Relativity

Lecture Notes and Course Overview

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Course idea. This course develops classical electrodynamics as one connected theory of sources, fields, geometry, and spacetime. We begin with static electric and magnetic fields. We then ask how local field laws are connected to fluxes and circulations on boundaries. This leads to the full Maxwell equations, electromagnetic waves, special relativity, radiation, and fields in matter. The goal is not to collect formulas. The goal is to understand why the formulas have the form they do, how they are connected, and how to use them in physical problems.

1. Main references and the design of these notes

Main references and intellectual framework

These lecture notes are written independently for this course, but they are built around two major sources.

1. **David J. Griffiths, *Introduction to Electrodynamics*, 4th edition.** This is the main undergraduate textbook reference. It provides the standard notation, core problems, and the level of calculation expected in the course. [Cambridge University Press page](#)
2. **Martin R. Zirnbauer, lecture notes on classical electrodynamics.** These notes are a major conceptual reference for the geometric structure of Maxwell theory, the relation between local and global laws, electric–magnetic duality, visualization, gauge structure, and the natural connection to relativity. Useful sources are:
 - [Zirnbauer’s official lecture-notes page](#),
 - *Classical Electrodynamics* (2011/12), Chapter 1: Maxwell in Chains,
 - *Classical Theoretical Physics II* (2015/16),
 - *Maxwell in Chains* (2016).

A useful supplementary reference is [David Tong, *Electromagnetism*](#).

The present notes do not copy the text or figures of these sources. The derivations, explanations, and illustrations are written for this course. When a structural viewpoint is especially close to one of the references, the source is stated explicitly.

The course keeps the standard undergraduate content close enough to Griffiths that students can use the textbook easily. At the same time, we use a stronger geometric viewpoint: charges and currents are sources, fields carry flux and circulation, boundaries connect local laws to global measurements, and relativity appears as part of the structure of Maxwell theory.

The advanced mathematical language used in some of Zirnbauer’s notes, such as differential forms and chain complexes, is **not required** in this course. We will use the useful physical ideas behind that language and explain them with ordinary vector calculus, simple geometry, and many figures.

2. The central question of electrodynamics

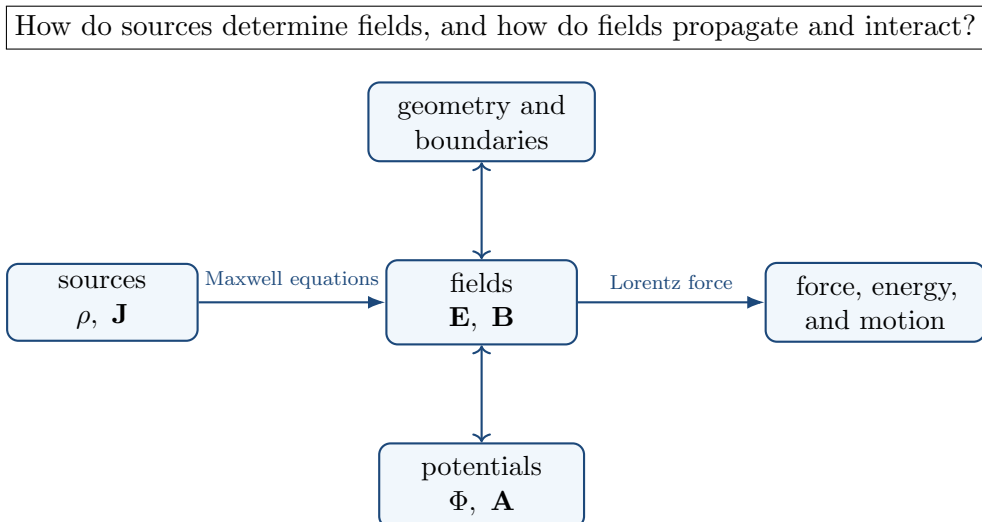
Electrodynamics studies how electric **charge** and electric **current** **create** electromagnetic fields, how these fields **act** on matter, and how the fields themselves **evolve** in space and time.

The main physical quantities are:

- $\rho(\mathbf{r}, t)$: electric charge density,
- $\mathbf{J}(\mathbf{r}, t)$: electric current density,
- $\mathbf{E}(\mathbf{r}, t)$: electric field,
- $\mathbf{B}(\mathbf{r}, t)$: magnetic field,
- $\Phi(\mathbf{r}, t)$: scalar potential,
- $\mathbf{A}(\mathbf{r}, t)$: vector potential.

Here $\mathbf{r}$ is position and t is time.

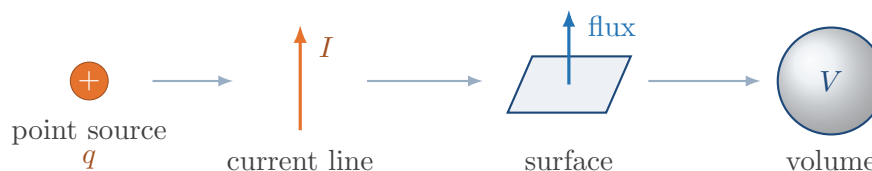
The whole course can be organized around the question



Sources determine the electromagnetic fields. Geometry and boundaries constrain their structure. Potentials provide a compact description of the fields. The fields then act on charges and matter.

3. Geometry first: points, lines, surfaces, and volumes

Electromagnetic sources, fields, and measurements naturally involve different geometric objects. This viewpoint is useful even if we use only ordinary vector calculus.



This picture is a way to remember geometric objects that repeatedly appear in the theory. For example, charge can be enclosed inside a volume, electric flux passes through a surface, and circulation is measured around a closed line. These geometric ideas will later become Gauss's law, Faraday's law, and the Ampère–Maxwell law.

Field lines and similar drawings are used only as visual aids. They are not physical strings in space.

4. Local laws and global laws

One of the most important ideas in the course is the relation between a **local statement** and a **global statement**.

A local law tells us what happens at one point. It uses divergence or curl. A global law tells us what happens over a finite surface, loop, or volume. It uses integrals. Gauss's and Stokes' theorems connect the two descriptions.

How to read the theory

We will repeatedly use the pattern

$$\boxed{\text{local law} \iff \text{global law}}$$

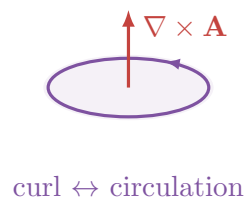
For divergence,

$$\int_V \nabla \cdot \mathbf{A} dV = \oint_{\partial V} \mathbf{A} \cdot d\mathbf{S}.$$

For curl,

$$\int_S (\nabla \times \mathbf{A}) \cdot d\mathbf{S} = \oint_{\partial S} \mathbf{A} \cdot d\mathbf{l}.$$

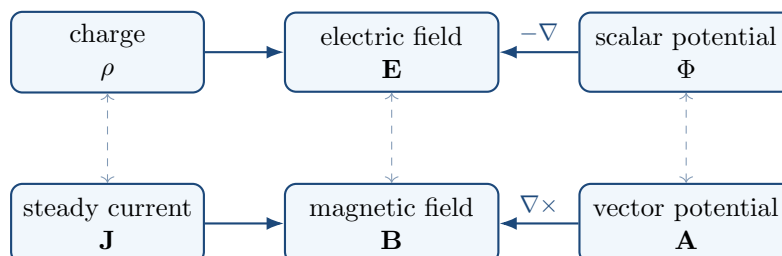
The first relation connects sources inside a volume to flux through its boundary. The second connects local circulation through a surface to circulation around its boundary.



This local–global structure will be a permanent visual motif in the lecture notes.

5. Electrostatics and magnetostatics as two parallel theories

The static part of the course is organized as two closely related theories.



In electrostatics,

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}, \quad \nabla \times \mathbf{E} = 0, \quad \mathbf{E} = -\nabla \Phi.$$

In magnetostatics,

$$\nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J}, \quad \mathbf{B} = \nabla \times \mathbf{A}.$$

The equations are not identical, but the comparison is extremely useful.

The reason electrostatics naturally uses a **scalar potential** is that the electrostatic field has zero curl,

$$\nabla \times \mathbf{E} = 0.$$

In a simply connected region, a curl-free vector field can be written as the gradient of a scalar function. Therefore,

$$\boxed{\mathbf{E} = -\nabla\Phi}$$

where Φ is a scalar potential.

By contrast, the magnetic field always has zero divergence,

$$\nabla \cdot \mathbf{B} = 0.$$

Locally, and globally under the usual topological and boundary conditions, a divergence-free vector field can be written as the curl of another vector field. Therefore,

$$\boxed{\mathbf{B} = \nabla \times \mathbf{A}}$$

where $\mathbf{A}$ is a vector potential.

So the different potentials are not arbitrary choices:

$$\boxed{\nabla \times \mathbf{E} = 0 \implies \mathbf{E} = -\nabla\Phi}$$

while

$$\boxed{\nabla \cdot \mathbf{B} = 0 \implies \mathbf{B} = \nabla \times \mathbf{A}.$$

This is why electrostatics is naturally organized by a scalar potential, while magnetostatics is naturally organized by a vector potential.

Later, in full time-dependent electrodynamics, both potentials are needed together:

$$\mathbf{B} = \nabla \times \mathbf{A}, \quad \mathbf{E} = -\nabla\Phi - \frac{\partial \mathbf{A}}{\partial t}.$$

This is the first step toward the unified Maxwell description.

6. Electrostatics: from charge to potential

Electrostatics asks how fixed charges create an electric field. We begin with Coulomb's law and superposition, then move to flux, Gauss's law, and the scalar potential.

Main topics are:

- Coulomb's law and superposition,
- electric field and electric flux,
- Gauss's law in global and local form,
- electric potential and conservative fields,
- Poisson and Laplace equations,
- conductors and boundary conditions,
- uniqueness and method of images,
- separation of variables,
- capacitance,
- multipole expansion and electric dipoles.

Gauss's law — charge is the source of the electric field

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}.$$

Electrostatic curl-free condition — electrostatic fields are conservative

$$\nabla \times \mathbf{E} = 0.$$

Electric potential relation

$$\mathbf{E} = -\nabla\Phi.$$

Poisson's equation — potential in the presence of charge

$$\nabla^2\Phi = -\frac{\rho}{\varepsilon_0}.$$

Laplace's equation — potential in a charge-free region

$$\nabla^2\Phi = 0.$$

Far from a localized charge distribution,

$$\Phi(\mathbf{r}) \simeq \frac{1}{4\pi\varepsilon_0} \left(\frac{Q}{r} + \frac{\mathbf{p} \cdot \hat{\mathbf{r}}}{r^2} + \cdots \right).$$

Here $Q = \int \rho(\mathbf{r}') d^3r'$ is the total charge and

$$\mathbf{p} = \int \mathbf{r}' \rho(\mathbf{r}') d^3r'$$

is the electric dipole moment. The multipole expansion tells us which features of a source remain visible far away.

7. Magnetostatics: the magnetic mirror

Magnetostatics asks how steady currents create magnetic fields. We deliberately develop it in parallel with electrostatics so that the similarities and differences remain visible. Main topics are:

- current and current density,
- Biot–Savart law,
- Ampère's law,
- magnetic flux,
- magnetic vector potential,
- gauge freedom,
- magnetic dipoles and magnetic moments,
- the electrostatic–magnetostatic structural analogy.

$$\nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J}, \quad \mathbf{B} = \nabla \times \mathbf{A}.$$

For a localized steady current distribution,

$$\mathbf{m} = \frac{1}{2} \int \mathbf{r}' \times \mathbf{J}(\mathbf{r}') d^3r',$$

where $\mathbf{m}$ is the magnetic dipole moment.

8. From static fields to Maxwell theory

The static equations are not the final theory. When **charges** and **currents** change with **time**, **electric** and **magnetic fields** become **coupled**.

Faraday induction tells us that a changing magnetic field produces a circulating electric field. Maxwell's correction to Ampère's law tells us that a changing electric field also contributes to magnetic circulation.

Faraday's law — changing **B** produces circulating **E**

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}.$$

Ampère–Maxwell law — current and changing **E** produce circulating **B**

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}.$$

Charge conservation is

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0.$$

The complete Maxwell equations are

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}, \quad \nabla \cdot \mathbf{B} = 0,$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}.$$

At this point the theory stops being two separate static theories. Electric and magnetic fields become one dynamical system.

9. Potentials, gauge freedom, and conservation laws

The potentials are not only convenient calculation tools. They reveal part of the structure of the theory.

$$\mathbf{B} = \nabla \times \mathbf{A}, \quad \mathbf{E} = -\nabla \Phi - \frac{\partial \mathbf{A}}{\partial t}.$$

A gauge transformation changes the potentials without changing the **physical fields**:

$$\mathbf{A} \rightarrow \mathbf{A} + \nabla \chi, \quad \Phi \rightarrow \Phi - \frac{\partial \chi}{\partial t}.$$

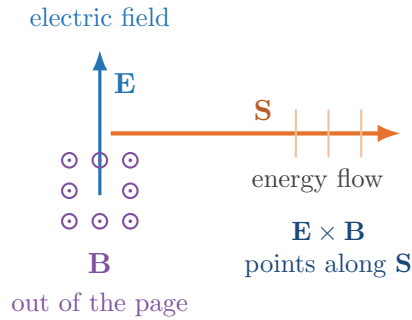
Here $\chi(\mathbf{r}, t)$ is an arbitrary sufficiently smooth scalar function.

We will also study energy and momentum carried by electromagnetic fields.

The electromagnetic field can store energy, but it can also **transport energy from one place to another**. In vacuum, the direction and rate of this energy transport are described by the **Poynting vector**,

$$\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}$$

where $\mathbf{S}$ is the **electromagnetic energy flux**: its direction gives the direction in which electromagnetic energy flows, and its magnitude gives the energy crossing unit area per unit time.



$\mathbf{B}$ points out of the page and $\mathbf{E}$ points upward, so the **right-hand rule** gives an energy flow $\mathbf{S}$ toward the right.

This gives an important physical picture: electromagnetic energy does not have to be carried by matter. It can be stored in the fields and can flow continuously through space.

Later, for electromagnetic waves, the same vector $\mathbf{S}$ will point in the direction in which the wave transports energy and momentum.

10. Maxwell theory naturally leads to relativity

Maxwell's theory contains an important surprise: electromagnetic disturbances propagate through vacuum with a fixed speed.

In vacuum, Maxwell's equations imply

$$\nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0, \quad \nabla^2 \mathbf{B} - \frac{1}{c^2} \frac{\partial^2 \mathbf{B}}{\partial t^2} = 0,$$

where

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}.$$

The speed c is therefore built directly into Maxwell's equations. Thus the speed of light is not added to electrodynamics by hand: it emerges from the electric and magnetic constants already present in Maxwell's equations. This immediately raises a physical question:

How can different inertial observers describe the same electromagnetic wave?

In Newtonian mechanics, two observers moving with relative speed v along the x direction use the Galilean transformation

$$x' = x - vt, \quad t' = t.$$

A velocity therefore transforms as

$$u' = u - v.$$

If a light signal has speed $u = c$ for one observer, Galilean kinematics would predict

$$u' = c - v$$

for another observer.

If the principle of relativity is to hold for electrodynamics, inertial observers must instead relate their measurements in a way that preserves the characteristic speed c appearing in Maxwell's equations.

Physical meaning

The central issue

Newtonian kinematics treats space and time separately. Maxwell theory points to a transformation that preserves the same electromagnetic wave speed c .

The solution is that space and time transform together.

For motion with speed v along the x direction, the **Lorentz transformation** is

$$x' = \gamma(x - vt), \quad t' = \gamma\left(t - \frac{vx}{c^2}\right),$$

with

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}.$$

The important point is already visible in these equations: x enters the transformation of time and t enters the transformation of position.

space and time are combined into spacetime

10.1 What does a light cone mean?

Consider a flash of light emitted at one event O ,

$$x = 0, \quad t = 0.$$

After a time t , light has travelled the distance

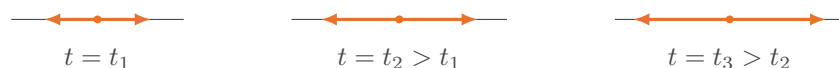
$$|x| = ct.$$

In one spatial dimension, the two light rays therefore satisfy

$$x = ct, \quad x = -ct.$$

The following picture first shows this in ordinary space.

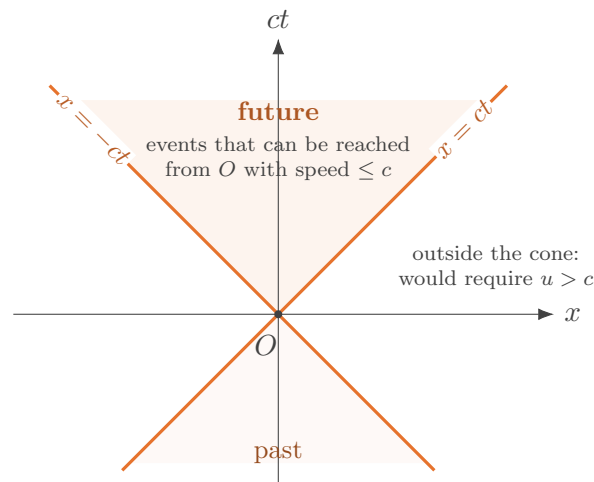
A flash of light expands outward at the fixed speed c .



As time increases, the light signal reaches a larger distance, but its boundary always satisfies $|x| = ct$.

A spacetime diagram shows the same process differently: position is drawn horizontally and time is drawn vertically.

We use ct instead of t on the vertical axis so that both axes have units of length. In this 1 + 1 dimensional picture, the two light-cone lines are a cross-section of the full light cone in spacetime.



The orange lines are the paths of light. Together they form the **light cone** of the event O .

The picture has a simple physical meaning:

- An event **inside the future light cone** can be reached from O by a signal moving with speed smaller than or equal to c .
- An event **on the light cone** is connected to O by light.
- An event **outside the light cone** would require a signal with speed greater than c .
- The **past light cone** contains events that could have influenced O .

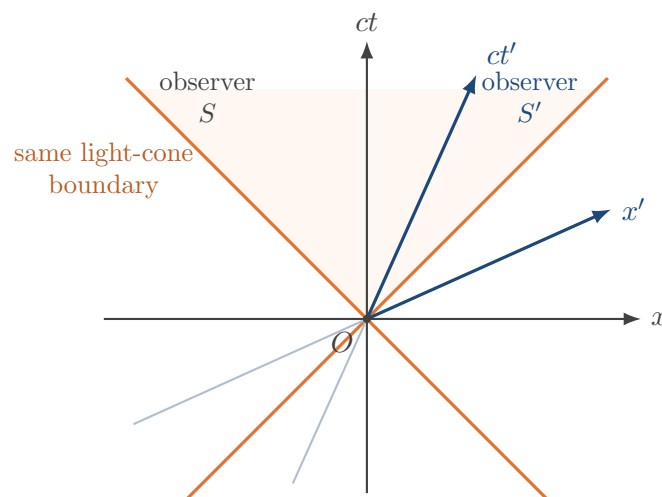
So the light cone gives the causal structure of spacetime.

10.2 Different observers use different space and time axes

Now consider another inertial observer S' moving with speed v along the positive x direction relative to observer S .

The Lorentz transformation changes the observer's space and time axes, but it leaves the light-cone directions unchanged.

The next figure shows only this geometric idea.



The observers use different space and time axes, but both describe the same light-cone directions.

For a boost with

$$\beta = \frac{v}{c},$$

the ct' axis satisfies

$$x = \beta ct,$$

while the x' axis satisfies

$$ct = \beta x.$$

The exact geometry will be developed later. For now, the important message is

different observers use different space and time coordinates,
but they share the same light-cone structure.

The invariant spacetime interval is

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2.$$

This gives the central conceptual chain:

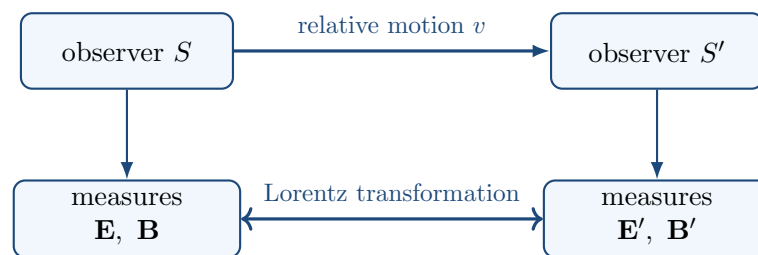
Maxwell equations $\rightarrow$ waves with a fixed speed c
 $\Downarrow$
 Galilean transformations are not sufficient
 $\Downarrow$
 space and time mix $\rightarrow$ Lorentz symmetry.

10.3 Electric and magnetic fields are parts of one field

Relativity also changes how we understand $\mathbf{E}$ and $\mathbf{B}$.

Two observers moving relative to one another generally measure different electric and magnetic fields.

The electric and magnetic components mix under a Lorentz transformation.



The same electromagnetic field is split into
electric and magnetic parts differently by different observers.

This is why electric and magnetic fields are naturally unified in relativistic electrodynamics.

The scalar potential Φ and vector potential $\mathbf{A}$ combine into the electromagnetic four-potential,

$$A^\mu = \left(\frac{\Phi}{c}, \mathbf{A} \right),$$

and the electric and magnetic fields are contained in the **electromagnetic field tensor**: $F^{\mu\nu}$,

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu.$$

At the roadmap level, the main message is simply

Maxwell theory contains the universal speed c ,
which leads naturally to spacetime and Lorentz symmetry,
and $\mathbf{E}$ and $\mathbf{B}$ become parts of one relativistic field.

Later in the course, we will develop these ideas quantitatively using Lorentz transformations, proper time, four-vectors, four-momentum, four-current, the four-potential, and the electromagnetic field tensor.

11. Waves, radiation, and causality

Maxwell's equations predict electromagnetic waves. We will study plane waves, polarization, reflection, transmission, energy flow, and momentum transport.

Maxwell's equations also determine the **geometry** of electromagnetic waves. For a plane wave in vacuum,

$$\mathbf{E} \perp \mathbf{B}, \quad \mathbf{E} \perp \mathbf{k}, \quad \mathbf{B} \perp \mathbf{k},$$

where $\mathbf{k}$ is the wave vector and gives the **direction of propagation** ($\mathbf{k} \parallel \mathbf{S}$). Thus electromagnetic waves are transverse: the electric and magnetic fields oscillate perpendicular to the direction in which the wave travels. This geometric structure will later lead naturally to polarization and energy transport.

Time-dependent sources also produce radiation. Because electromagnetic information propagates at finite speed, the field observed now depends on the source at an earlier time. The retarded time is

$$t_r = t - \frac{|\mathbf{r} - \mathbf{r}'|}{c}.$$

This relation is a direct statement of causality: a change at the source cannot influence a distant point instantly.

We will study retarded potentials, near and radiation zones, electric dipole radiation, angular radiation patterns, and radiated power.

12. Electromagnetism in matter

The final part of the course asks how materials respond to electromagnetic fields. Instead of following every microscopic charge separately, we introduce macroscopic fields that summarize the response of matter.

We will study polarization $\mathbf{P}$, magnetization $\mathbf{M}$, the displacement field $\mathbf{D}$, and the auxiliary magnetic field $\mathbf{H}$.

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}, \quad \rho_b = -\nabla \cdot \mathbf{P},$$

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M}.$$

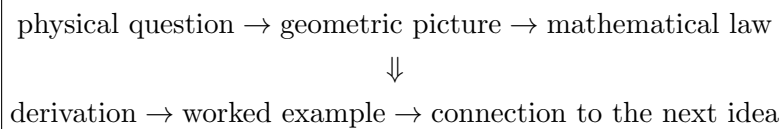
For simple linear media,

$$\mathbf{D} = \varepsilon \mathbf{E}, \quad \mathbf{B} = \mu \mathbf{H}.$$

Boundary conditions become especially important here because fields must be matched correctly across material interfaces.

13. How each lecture note is organized

To keep the course readable for students meeting the material for the first time, each week follows the same logic.



The notes use several repeated visual elements:

- **Key relations:** the small set of equations that organize the topic,
- **Physical meaning:** the geometric or physical interpretation of an equation,
- **Derivation:** a complete derivation in smaller gray text,
- **Examples:** direct applications to electrodynamics,
- **Local ↔ global:** the relation between differential and integral laws,
- **Figures:** simple colored diagrams that make direction, orientation, flux, circulation, and symmetry visible.

14. Week-by-week roadmap

The exact timing may change slightly, but the course will roughly follow this plan.

Week	Main focus
1	Mathematical language of fields: vectors, scalar and vector fields, gradient, divergence, curl, Laplacian, line/surface/volume integrals, Gauss and Stokes theorems, coordinate systems, and the Dirac delta. The geometric meaning of sources, flux, circulation, and boundaries is introduced from the start.
2	Electrostatics from sources to fields: Coulomb's law, superposition, electric field, electric flux, Gauss's law in integral and differential form, symmetry methods, conservative fields, and electric potential.
3	Potential theory and boundaries: Poisson and Laplace equations, boundary conditions, uniqueness, conductors, and the method of images. The focus is on how field equations become boundary-value problems.
4	Solving electrostatic boundary-value problems: separation of variables, capacitance, electrostatic energy, multipole expansion, and electric dipoles.
5	Magnetostatics as the magnetic mirror of electrostatics: current density, Biot–Savart law, Ampère's law, magnetic flux, symmetry, and the Lorentz force.
6	Magnetic vector potential and structure: $\mathbf{B} = \nabla \times \mathbf{A}$, gauge freedom, magnetic dipoles, magnetic moments, and the structural relation between electrostatics and magnetostatics.
7	From statics to dynamics: Faraday induction, electromotive force, charge conservation, the Ampère–Maxwell law, and the complete Maxwell equations in local and global form.
8	Potentials and conservation laws: time-dependent scalar and vector potentials, gauge transformations, Coulomb and Lorenz gauges, electromagnetic energy, Poynting theorem, momentum, and stress.
9	Maxwell waves and the road to relativity: wave equations, the speed c , plane-wave structure, and why Maxwell theory requires a new relation between space and time. Lorentz transformations and spacetime are introduced from this physical motivation.
10	Relativistic electrodynamics: proper time, four-vectors, four-momentum, four-current, four-potential, transformation of $\mathbf{E}$ and $\mathbf{B}$, electromagnetic field tensor, and covariant Maxwell equations.
11	Electromagnetic waves and optics: monochromatic waves, polarization, energy and momentum transport, reflection, transmission, and optical consequences of Maxwell's equations.
12	Radiation begins: retarded potentials, causality, near and far fields, and the physical distinction between static, induction, and radiation fields.
13	Electromagnetic radiation: electric dipole radiation, angular patterns, radiated power, and radiation from accelerated charges when time allows.
14	Electromagnetism in matter: polarization, bound charge, magnetization, bound current, $\mathbf{D}$, $\mathbf{H}$, constitutive relations, boundary conditions, and waves in media.

15. What you should be able to do by the end

By the end of the course, you should be able to:

- translate a physical electromagnetic problem into the correct field equations,
- use symmetry before starting a long calculation,
- move between local and global forms of Maxwell's laws,
- solve standard electrostatic and magnetostatic problems,
- use scalar and vector potentials and explain gauge freedom,
- derive and interpret electromagnetic wave equations,
- explain why Maxwell theory leads naturally to special relativity,
- use four-vectors and the electromagnetic field tensor at an introductory level,

- calculate basic radiation fields and radiated power,
- describe electromagnetic response in simple materials,
- explain the geometry and physical meaning behind the equations, not only manipulate them.

16. Assessment

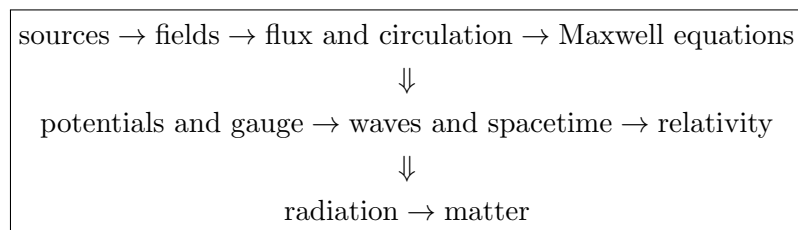
Examination:	Written examination
Duration:	120 minutes
Weight:	100%
Minimum for passing:	45%

Homework problems are an essential part of the course. They are designed to turn the geometric and conceptual picture into calculation skills.

17. The whole course at a glance

Part	Physical question	Main mathematical structure
Mathematical language	How do we describe fields and their geometry?	Gradient, divergence, curl, Laplacian, boundaries, integral theorems.
Electrostatics	How do fixed charges create electric fields?	Coulomb law, flux, Gauss law, scalar potential, Poisson/Laplace equations.
Magnetostatics	How do steady currents create magnetic fields?	Biot–Savart, Ampère law, magnetic flux, vector potential.
Maxwell dynamics	How do changing fields create one another?	Faraday law, Ampère–Maxwell law, continuity equation.
Potentials and gauge	How can the same physical field have different potential descriptions?	Φ , $\mathbf{A}$, gauge transformations, Lorenz gauge.
Relativity	How do different inertial observers describe the same electromagnetic field?	Lorentz transformations, spacetime, four-vectors, field tensor.
Waves	How does the electromagnetic field propagate?	Wave equations, polarization, energy and momentum flow.
Radiation	How do time-dependent sources send energy to large distances?	Retarded potentials, far fields, dipole radiation.
Matter	How do materials respond to fields?	Polarization, magnetization, $\mathbf{D}$, $\mathbf{H}$, constitutive relations.

The main conceptual path is



The course starts with concrete fields around charges and currents. It ends with one unified picture in which geometry, dynamics, relativity, radiation, and matter all belong to the same electromagnetic theory.